A Note on Private Contribution towards Multiple Public Goods

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Abstract

In this note we review the contribution pattern predicted by Ghosh, Karaivanov and Oak (2007) in their model of 'separate provision scheme' towards voluntary provision of multiple pure public goods. We replace their example of Cobb-Douglas preference, which is very special one, with a more standard specification of such preference and show unlike their prediction, independent of homogeneous or heterogeneous preferences there exists at least a range of income inequality where the contribution levels of the individuals become indeterminate. Our result finds another reason to support the conclusion of Ghosh, Karaivanov and Oak (2007) that 'joint provision scheme' is more efficient than the 'separate provision scheme'.

Keywords: public goods, inequality, income distribution.

JEL Classification: H41, D63, D31.

1. Introduction

In the case of private provision of multiple pure public goods, Ghosh, Karaivanov and Oak (2007) compare a 'separate provision scheme' where individuals contribute separately for each of the public goods with a 'joint provision scheme' where the individuals contribute towards a 'bundle' of public goods. To substantiate their findings, they use an example of Cobb-Douglas preference, where the sum of willingness to spend for these commodities exceeds 1. This assumption is very special one because in standard static theory of consumer behavior, the 'willingness to pay' parameters in the Cobb-Douglas utility functions are interpreted as the share of her budget an individual is willing to spend on the commodities in the consumption basket and therefore the sum of the parameters is always equal to 1. As we adopt this standard example of Cobb-Douglas utility function to rework their exercise, to our surprise, we find that in case of separate provision scheme and independent of homogeneous or heterogeneous preferences there exists at least a range of income inequality where the contribution levels of the individuals become indeterminate[1]. The apprehension about the indeterminacy was expressed earlier also by Cornes and Itaya (2004). Since this problem does not arise in the case of single pure public good (Bergstrom, Blume and Varian (1986)), they attributed it to the 'coordination' problem which specially arises in the case of multiple public goods. It was surprising that Ghosh, Karaivanov and Oak (2007) did not obtain this indeterminacy. With a standard Cobb-Douglas

[1] In fact we obtain the indeterminacy even with the specification used by Ghosh, Karaivanov and Oak (2007).

specification, we find that depending on the case of homogeneous and heterogeneous preferences, indeterminacy occurs in particular ranges of income distribution. We derive the exact range of income inequality where the indeterminacy occurs. Rather than weakening the result obtained by Ghosh, Karaivanov and Oak (2007), our result adds another reason to support the conclusion of their paper that 'joint provision scheme' is more efficient than the 'separate provision scheme' in the case of voluntary provision of multiple public goods.

The following section presents the model and then it concludes.

2. The Model

We consider an economy consisting of two individuals. Individual 1 is rich with an income level M_1 and individual 2 is poor with income level $M_2 < M_1$. Two distinct pure public goods are produced in the economy, the amounts of which are represented as G and H respectively. Both the individuals derive utilities from consumption of public goods and a private good. The public goods are completely financed by voluntary contributions of the individuals. We assume that \$1 of contribution can be converted into 1 unit of public good. Under 'separate provision scheme' the individuals contribute towards each of the two public goods separately. So if i^{th} individual (for all i = 1, 2) contributes g_i and h_i for G and H respectively, the total amount of public good G provided is (g_1+g_2) and the same for H is (h_1+h_2). The amount spent by the i^{th} individual on the private good is denoted by c_i. We assume all the goods are normal in their consumption and their prices are unity.

2.1 The Equilibrium under Homogeneous Preference

We assume the preference pattern of the i^{th} individual is given by the utility function,

$$U_i = \alpha lnG + \beta lnH + \gamma lnc_i \qquad \forall i = 1, 2 \qquad (1)$$

where $\alpha, \beta, \gamma \in (0, 1)$ are the preference parameters and $\alpha+\beta+\gamma=1$. Now without loss of any generality we assume $\beta > \alpha$ [2] which implies both the individuals prefer public good H more than G. Note the only difference of the specification used in equation (1) from Ghosh et al (2007) that they assume $\gamma = 1$ instead of $\gamma < 1$ as we have done. It is implicit in Ghosh et al (2007) that $\alpha+\beta+\gamma > 1$, which is unusual because of the natural interpretation of the taste parameters α, β, γ. In standard static theory of consumer behavior, they are interpreted as the share of her budget the individual is willing to spend on the commodities in the consumption basket[3]. So in equation (1) we rectify the problem and go back to the more standard specification of Cobb-Douglas utility function.

Since $G = g_1+g_2$ and $H = h_1+h_2$, equation (1) can alternatively be written as:

$$U_i = \alpha ln(g_i+g_j) + \beta ln(h_i+h_j) + \gamma lnc_i \qquad \forall i, j=1,2;\ i \neq j \qquad (2)$$

[2] Ghosh et al (2007) also use the similar preference specification.

[3] See Varian (1993).

How do individuals determine the values of g_1, g_2, h_1 and h_2?

The i^{th} individual maximizes the value of her utility given in equation (2) by choosing (g_i, h_i) subject to her budget constraint $M_i \geq g_i + h_i + c_i$. Since this is a static model, there is no incentive for savings at equilibrium. It follows that the equilibrium value of c_i must be chosen in such a way that $c_i = M_i - g_i - h_i$. Using this fact individual i's problem can be rewritten in the following way:

Maximization of $Ui = \alpha ln(g_i+g_j) + \beta ln(h_i+h_j) + \gamma ln(M_i - g_i - h_i)$ with respect to $g_i \geq 0$ and $h_i \geq 0$ $\forall$ $i, j=1,2; i \neq j$.

The Kuhn-Tucker conditions of utility maximization for individual 1 and 2 turn out as:

$$\frac{\alpha}{g_1+g_2} \leq \frac{\gamma}{M_1-g_1-h_1} \quad \text{(for } g_1 \geq 0\text{)} \tag{3}$$

$$\frac{\beta}{h_1+h_2} \leq \frac{\gamma}{M_1-g_1-h_1} \quad \text{(for } h_1 \geq 0\text{)} \tag{4}$$

$$\frac{\alpha}{g_1+g_2} \leq \frac{\gamma}{M_2-g_2-h_2} \quad \text{(for } g_2 \geq 0\text{)} \tag{5}$$

$$\frac{\beta}{h_1+h_2} \leq \frac{\gamma}{M_2-g_2-h_2} \quad \text{(for } h_2 \geq 0\text{)} \tag{6}$$

We take $\frac{M_2}{M_1} \in [0,1]$ as the index of income inequality. So using the Kuhn-Tucker conditions (3) – (6), the Nash equilibrium of the voluntary contribution game can be characterized as we do it in Proposition 1 below.

Proposition 1: *Under Separate Provision with homogeneous preference (1) if* $\frac{M_2}{M_1} < \gamma$ *i.e. for sufficiently high income inequality only the rich contributes for both the public goods. The rich contributes* αM_1 *for G and* βM_1 *for H. (2) No equilibrium exists where only the poor contributes for both the public goods. (3) Even if equilibrium exists for sufficiently low income inequality i.e.* $\gamma < \frac{M_2}{M_1} < 1$ *where both the individuals contribute positive amounts, the amounts are indeterminate.*

Proof: See the Appendix.

If the income inequality is high enough only the rich contributes for the public goods and the poor enjoys the benefit of the public goods without the contribution. Since the rich individual values the public goods and has ability to pay for it, she contributes even if the poor free rides. She contributes for each of the two public goods according to her valuation of the goods. But, in a more equal society the problem of coordination failure pops up. This is the reflection of the possibility of indeterminacy in more than one public goods model as mentioned by Cornes and Itaya (2004). We show that this is more of a problem in a society with low income inequality.

2.2. The Equilibrium under Heterogeneous Preference

In this case we write the utility functions of the individuals 1 and 2 as:

$$U_1 = \alpha \ln G + \beta \ln H + \gamma \ln c_1 \quad (7)$$

$$U_2 = \beta \ln G + \alpha \ln H + \gamma \ln c_2 \quad (8)$$

Here α, β and γ have the same interpretations as under homogeneous preference case with $\alpha, \beta, \gamma \in (0,1)$ and $\alpha + \beta + \gamma = 1$. Note that in this case $\beta > \alpha$ means that the rich has more liking towards the public good H than the poor.

Proceeding similarly as for homogeneous preference we find a range of moderate income inequality in the case of heterogeneous preference where both the individuals contribute for both the public goods but their contribution amounts become indeterminate. This again confirms the existence of "coordination problem" as observed by Itaya et al (2004) even for heterogeneous preference pattern.

Proposition 2: *Under Separate Provision with heterogeneous preference, (1) only the rich contributes to both the public goods if there is high degree of income inequality i.e. if* $0 < \frac{M_2}{M_1} < \frac{\alpha}{\beta}\gamma$. *The rich contributes* αM_1 *for G and* βM_1 *for H. (2) When the degree of income inequality is low i.e.* $\frac{\alpha}{\beta} < \frac{M_2}{M_1} < 1$ *the rich contributes* $\frac{\beta M_1}{1-\alpha}$ *for H and the poor contributes* $\frac{\beta M_2}{1-\alpha}$ *for G. (3) Otherwise when* $\frac{\alpha}{\beta}\gamma < \frac{M_2}{M_1} < \frac{\alpha}{\beta}$ *as both the individuals contribute for both the public goods, the contribution levels become indeterminate.*

Proof: The proof is similar to the proof of proposition 1.

3. Conclusion

This note considers an economy where the individuals contribute voluntarily for the provision of multiple pure public goods through 'separate provision scheme'. Using a utility function which is a slight variation of the Cobb-Douglas example discussed by Ghosh et al (2007) and closer to the standard consumer behavior theory, we study the effect of change in income inequality on the amount contributed. We observe both with the homogeneous and heterogeneous preferences, indeterminacy of contribution amounts occurs at some ranges of income inequality. This observation confirms the existence of "coordination problem" as mentioned by Cornes and Itaya (2004) in the case voluntary provision of multiple public goods. It also adds another reason to support the conclusion drawn by Ghosh et al (2007) that 'separate provision scheme' is more efficient than the 'joint provision scheme' in the case of voluntary provision of multiple public goods.

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Appendix

Proof of Proposition 1: Type I Equilibrium ($g_1 > 0, h_1 > 0, g_2 = 0, h_2 = 0$)

Since ($g_1 > 0, h_1 > 0$) from (3) and (4) it follows:

$$(\alpha+\gamma)g_1 + \alpha h_1 = \alpha M_1 \quad \text{(A1)}$$

$$\beta g_1 + (\beta+\gamma)h_1 = \beta M_1 . \quad \text{(A2)}$$

Solving (A1) and (A2) we get: $g_1 = \alpha M_1$ and $h_1 = \beta M_1$.

Substituting the values of g_1, g_2, h_1 and h_2 in equations (5) and (6) we get the necessary condition for the existence of this equilibrium as $\frac{M_2}{M_1} < \gamma$.

Type II Equilibrium ($g_1 = 0, h_1 = 0, g_2 > 0, h_2 > 0$)

Since ($g_2 > 0, h_2 > 0$) from (5) and (6) it follows that the following must be true:

$$(\alpha+\gamma)g_2 + \alpha h_2 = \alpha M_2 \quad \text{(A3)}$$

$$\beta g_2 + (\beta+\gamma)h_2 = \beta M_2 . \quad \text{(A4)}$$

Solving (A3) and (A4) we get: $g_2 = \alpha M_2$ and $h_2 = \beta M_2$.

Substituting the values of g_1, h_1, g_2 and h_2 in equations (3) and (4) we find the necessary condition for the existence of this equilibrium as $\gamma > \frac{M_1}{M_2}$. But, since $\frac{M_1}{M_2} > 1$ and $\gamma < 1$, the necessary condition never holds.

Type III Equilibrium ($g_1 > 0, h_1 > 0, g_2 > 0, h_2 > 0$)

Since ($g_1 > 0, h_1 > 0, g_2 > 0, h_2 > 0$) from equations (3) - (6) it follows that the following must be true:

$$(\alpha+\gamma)g_1 + \alpha h_1 + \gamma g_2 = \alpha M_1$$
$$\beta g_1 + (\beta+\gamma)h_1 + \gamma h_2 = \beta M_1$$
$$\gamma g_1 + (\alpha+\gamma)g_2 + \alpha h_2 = \alpha M_2$$
$$\gamma h_1 + \beta g_2 + (\beta+\gamma)h_2 = \beta M_2$$

But, as we solve these equations, we find that the coefficient matrix of the above system of equations is singular i.e. the corresponding determinant is zero. This implies that all the equations are not linearly independent. Such a linearly dependent non-homogeneous system of equations can only yield infinite combinations of g_1, h_1, g_2 and h_2. Therefore, in this equilibrium the amounts of contribution are indeterminate.